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Large two-dimensional sonic band gaps

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We show that absolute sonic band gaps produced by two-dimensional square and triangular lattices of rigid cylinders in air can be increased by reducing the structure symmetry. In the case of square lattices, symmetry reduction is achieved by a smaller diameter cylinder placed at the center of each unit cell. For triangular lattices the reduction is achieved by decreasing the diameter of the cylinder at the center of the hexagons in the lattice. Theoretical predictions are also demonstrated experimentally: starting from a honeycomb lattice (using cylinders of 4 cm of diameter size and 6.35 cm nearest-neighbor distance) we have studied the transition to a triangular symmetry by putting rods with increasing diameter (in the range 0.6–4 cm) at the center. The greatest enhancement of the attenuation strength observed in transmission experiments has been obtained in the high frequency region for diameter ratios in the range 0.1–0.3. [S1063-651X(99)51212-9]

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Since the seminal work of Yablonovitch [1] and John [2], who open the research in photonic crystals (PC), we have witnessed the appearance of new devices based on the existence of photonic band gaps (PBG) in these dielectric periodic structures. Also, because the underlying theory is applicable to other kind of waves, acoustic and elastic, a search for periodic structures having properties of sonic band gap (SBG) or elastic band gap (EBG), respectively, is currently being performed. Therefore, the design, construction, and technological applications of a completely new type of crystals that could be called classical wave crystals is becoming a very promising project at the beginning of the new century.

In the field of acoustics much theoretical work has been done proposing structures having SBG's properties [3–6]. The existence of SBG is due to a complex interplay between the sound velocity and density ratios of the composite materials, and their spatial arrangement. Experimentally, a few works have claimed to observe absolute band gaps [7,8]. Therefore, the actual possibility of building up composite material having SBG open a new technological research on environmental protection.

One of the goals of PBG and SBG theory is the search of materials and/or topologies producing large gaps at the desired range of frequencies. In PC it has been shown that full PBG can be enlarged by decreasing the crystal symmetry through the introduction of a two-point basis set. Thus, for example, 3D PC based on a face-centered-cubic structure do not possess a full PBG between the first and second bands, but the ones having a diamond structure do because the additional point basis lifts the degeneracy of some bands [9]. In the same manner, in two-dimensional (2D) structures, Anderson and Giapis [10] obtained larger gaps when they add a different size rod at the center of each unit cell of square and honeycomb lattices. The latest conclusion awakened our interest about whether this sort of mechanism is effective in creating large SBG and/or enhancing the attenuation measured in some 2D sonic crystals (SC) previously studied by us [8]. In this Rapid Communication we show that this mechanism is also effective in SC, being the cause for such SBG enlargement a combination of two mechanism: (i) the achievement of higher filling fractions, and (ii) the symmetry reduction. Also, here we demonstrate that mechanism (ii) is much more effective than mechanism (i).

The wave equation for the propagation of pressure waves $p(\mathbf{r})$, with harmonic frequency, ω, in a 2D-space $\mathbf{r}=(x,y)$ defined by a composite system having a sound velocity, $v(\mathbf{r})$, and density, $\rho(\mathbf{r})$, can be written as

$$\boldsymbol{\nabla}\cdot\left(\frac{\boldsymbol{\nabla}p(\mathbf{r})}{\rho(\mathbf{r})}\right)=-\frac{\omega^2}{v^2(\mathbf{r})\rho(\mathbf{r})}p(\mathbf{r}). \qquad (1)$$

This equation closely resembles to that of transverse-magnetic waves propagating through a 2D composite dielectric medium $\epsilon(\mathbf{r})$. The corresponding magnetic field, $\mathbf{H}=H(\mathbf{r})\hat{\mathbf{r}}$, verifies the equation

$$\boldsymbol{\nabla}\cdot\left(\frac{\boldsymbol{\nabla}H(\mathbf{r})}{\epsilon(\mathbf{r})}\right)=-\frac{\omega^2}{c^2}H(\mathbf{r}), \qquad (2)$$

where c is the light velocity. The resemblance between Eqs. (1) and (2) suggests that the results found in Ref. [10] could be extended to sound waves. Nevertheless, extrapolation of results obtained on PBG theory to SC can be misleading because of the different magnitudes involved in both equations. Also, the different kind of waves involved (vectorial in PC and scalar in SC), the position dependence of the bulk modulus $\lambda(\mathbf{r})=v^2(\mathbf{r})\rho(\mathbf{r})$, and the huge contrast between material parameters in the SC under research requires that

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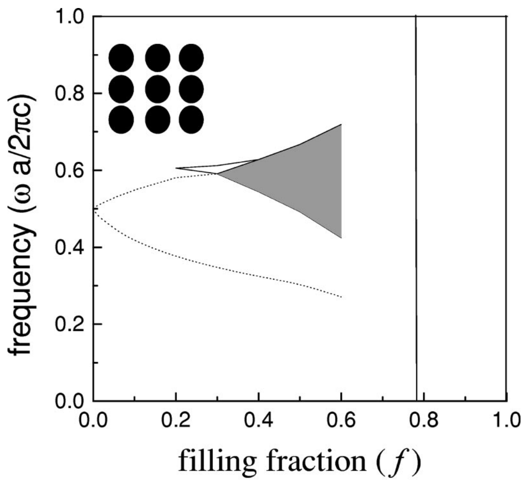

FIG. 1. Gap map for a square lattice of rigid cylinders in air. The dotted (solid) line defines the limits of the first pseudogap along the ΓX (ΓM) direction. An absolute sonic band gap (shaded zone) occurs when the gaps along the two high symmetry directions overlap. The maximum gap will be produced at the closed-packing condition, $f(\mathrm{CP})=0.78$, which is indicated by the vertical line.

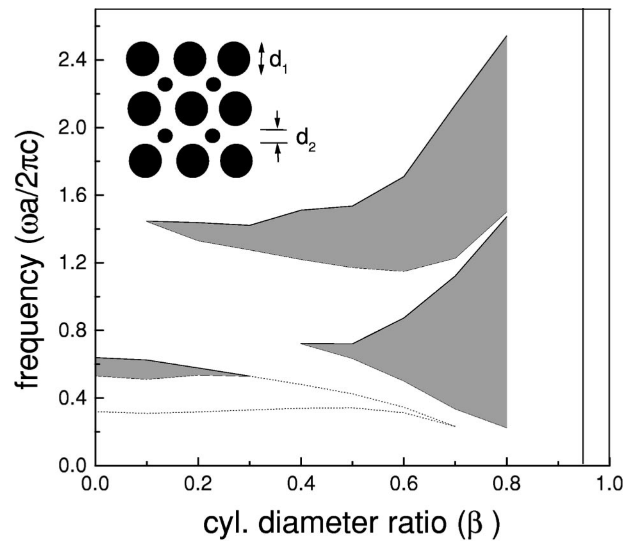

FIG. 2. Gap map for the reduced-symmetry square lattice of rigid cylinders in air. The filling fraction $f=0.415$ at $\beta=0$. For $\beta \geqslant 0.1$ the gaps are significatively larger than the ones of the square lattice with equal f (see Fig. 1). The maximum gap will be achieved at the close-packing condition, $\beta(\mathrm{CP})=0.94$, which is defined by the vertical line.

the conclusions of Anderson and Giapis [10] for PC must be confirmed both theoretically and experimentally in SC.

We have obtained the acoustic band structure by solving Eq. (1) using a variational method introduced by some of us [11]. The pressure is developed as a linear combination of localized waves and the differential problem is transformed into a matrix problem. A brief report of the method can be found elsewhere [8]. Here the results were obtained using 225 localized functions.

With regard to experiments, they have been performed by using the same set up as described in Ref. [8]. Briefly, in a echo-free chamber, we study how the sound is scattered by different samples, which resemble minimalist sculptures [12]. The sound transmitted across these structures is recorded by a sample microphone and is compared to that recorded by a reference microphone (which received the sound without any attenuation by the sample). Each sample consists of 1 m long aluminum bars hanging on a frame which can rotated around the vertical axis to explore any direction of the **k** wave vector perpendicular to the cylinders axis. Thus, to analyze the honeycomb-triangular transition, cylinders with diameter $d_1=4$ cm are initially arranged in a honeycomb lattice, being the nearest neighbor distance 6.35 cm. Afterwards, an additional cylinder with a variable diameter size d_2 (between 0.6 and 4 cm) is placed at the center of each hexagon in the starting lattice. Apart from the zero-order transmission experiments (where the source and sample microphone are colinear) we have also investigated possible energy transfer to Bragg waves of higher orders by putting the sample microphone at angles tilted with respect to the ΓX or ΓJ direction of the Brillouin zone (BZ).

First, we start with the square lattice whose properties have been studied by different groups [8,12–15]. Figure 1 summarizes the result of our calculations for this lattice. The gap map shows that both directions of high symmetry have pseudogaps that overlap each other for filling fractions, $f \geqslant 0.3$. The maximum absolute gap occurs when the cylinders are close-packed, $f(\mathrm{CP})=0.78$. We have modified this structure by adding a smaller rigid cylinder at the center of each square unit cell. Although the point group symmetry of the Bravais lattice (C_{4v}) does not change, the new crystal structure has lower symmetry since the unit cell contains two distinct basis units with no symmetry inversion. The addition of the new cylinder, of diameter d_2, at the center of the lattice enhances the filling fraction, which takes the value $f_r=\pi(d_1^2+d_2^2)/(4a^2)$. If the starting square lattice has a filling fraction $f \leqslant 0.39$ the diameter d_2 can be increased up to d_1, and therefore, the gap of a square lattice with periodicity $a/\sqrt{2}$ is recovered. On the other hand, when the starting lattice has $0.39 \leqslant f \leqslant f(\mathrm{CP})$, d_2 is limited by the close-packing condition; $d_{2,max}=\sqrt{2}a-d_1$. In this case, one can obtain maximun f_r when the close-packing condition is fulfilled, $f_{r,max}(\mathrm{CP})=0.92$. After extensive calculation with this structure, we found a general behavior. The gap between the first and the second bands (if it exists) decreases when we increase the diameter ratio ($\beta=d_2/d_1$) due to the bands approaching at the M point of the BZ [the (110) point]. At the same time, new gaps appear at higher frequencies. In Fig. 2 we show this effect for the case in which a square lattice with $f=0.415$ get additional gaps by using the symmetry reduction mechanism.

In the case of triangular lattices (their point group is C_{3v}), the gap map (not shown here) presents the first absolute gap (the one between the first and second band) at $f \geqslant 0.5$ [8]. For the honeycomb lattice (a reduced-symmetry lattice of the triangular), its gap map is plotted in Fig. 3. The honeycomb lattice always has larger gaps than the triangular one provided both lattices have equal f values. This is an example of how symmetry reduction produce and enlarge the absolute gaps.

We have measured the evolution of the gaps when the honeycomb lattice is transformed into a triangular lattice by increasing the diameter, d_2, of the cylinder placed at the center of each hexagon. We started with the structure having

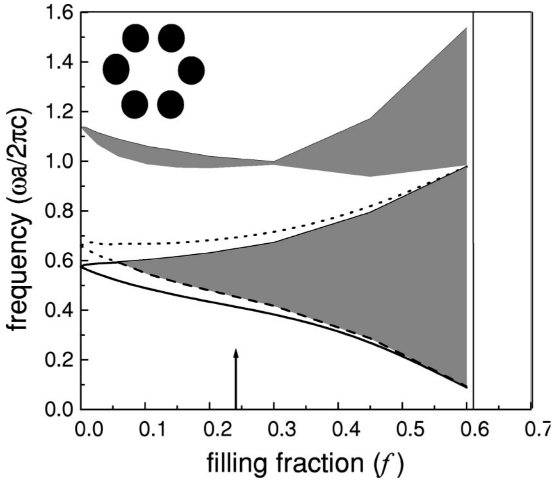

FIG. 3. Gap map for the honeycomb lattice of rigid cylinders in air. The first absolute gap is defined by the overlap between the pseudogaps at the *X* point (whose limits are represented by the dotted lines), and at the *M* point (the continuous lines define its limits). The vertical line defines the close-packing condition in this lattice, $f(\mathrm{CP})=0.605$.

$f=0.24$, which is indicated by an arrow in Fig. 3. Experimental details have been explained previously. In Fig. 4 we display typical attenuation spectra taken along the two high symmetry direction of the BZ. Figure 5 shows the comparison between the absolute gaps calculated as a function of β and the borders of the attenuation band measured in transmission experiments (e.g., the arrows in Fig. 4). It is remarkable how the magnitude of the first gap decreases although f increases. The reason for this behavior is the enhancement of the symmetry as β increases. The full symmetry is finally achieved at $\beta=1$, the triangular lattice condition. Also, notice the agreement between the experimentally deduced borders of the low frequency gap and the calculated one. The disagreement observed for the high frequency gap can be

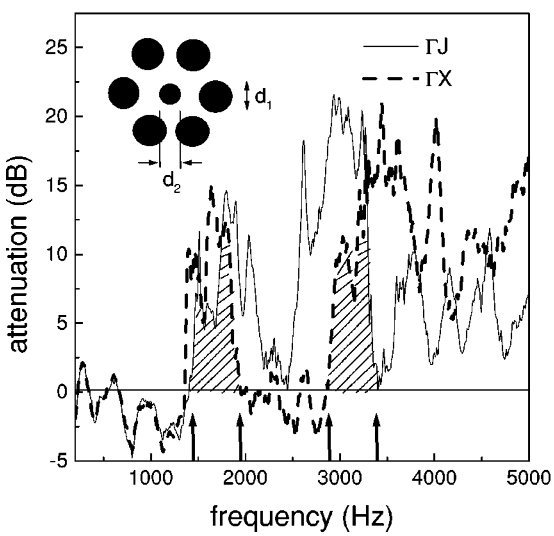

FIG. 4. Attenuation spectra taken along the ΓJ and ΓX directions, for the case of a reduced-symmetry triangular ($d_1=4$ cm, and $d_2=1$ cm). The first two overlaps between attenuation bands are the hatched zones in the plot and their borders (indicated by the arrows) are plotted in Fig. 5. as full circles.

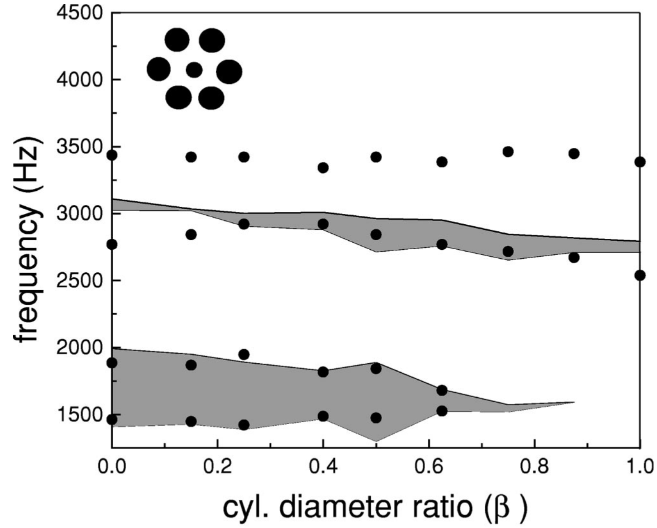

FIG. 5. Gap map for the reduced-symmetry triangular lattice of rigid cylinders in air. The shaded areas represent the calculated gap. The dots define the limits of the overlap between attenuation bands observed in transmission spectra measured along ΓX and ΓM, respectively.

understood taking into account two other mechanisms that can produce attenuation bands in zero-order transmission experiments: (1) the existence of deaf bands [8], and (2) an energy transfer to Bragg waves of higher orders. The latest mechanisms have been experimentally explored through the detection of the scattered sound along a direction different from the incident *k* wave vector. We have found no conclusive results supporting that this mechanism is the only one responsible of such big attenuation bands. In addition, the analysis of the pressure pattern of states in the bands existing on that frequency range shows that they are antisymmetric regarding the plane defined by the incident wave-vector and the cylinder axis, and this property is the fingerprint of deaf states. Therefore, we conclude that attenuation bands in the frequency region 2700–3500 Hz are mainly due to deaf bands.

In soundshielding devices not only the frequency range of gaps is important but also the strength of attenuation ob-

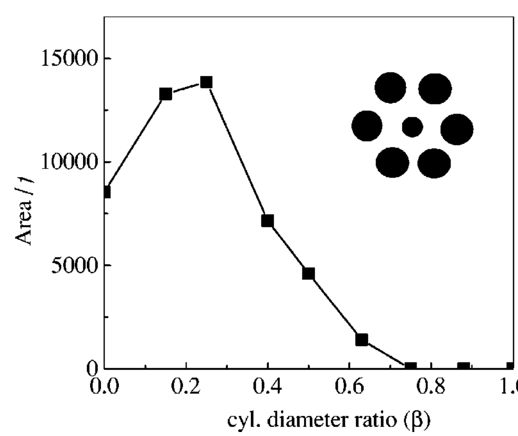

FIG. 6. Area, normalized to the filling fraction *f*, of the first attenuation-band-overlap observed in transmission pectra taken along the ΓJ and ΓX directions, in the reduced-symmetry honeycomb lattice. The line is a guide for the eye.

tained in the sound transmitted. In Fig. 6 we plot the area of the first attenuation band normalized to the filling fraction as a function of β for the honeycomb-triangular transition studied in Fig. 5. This normalized area could be considered as a single parameter defining the attenuation strength of a given sonic structure. We observe that the attenuation strength is maximum when $\beta \approx 0.2$. This result lets us conclude that in order to improve soundproofing devices it is preferable to sacrifice a little of band-gap-width in order to have stronger attenuation effects.

In summary, we have shown that symmetry reduction in 2D square and triangular sonic crystals result in a band-gap enlargement as in photonic crystals. Experiments performed on the honeycomb-triangular transition confirm our theoretical findings with regards to the behavior of the low frequency band gap. For the high frequency gap, our theoretical analysis and experimental characterization indicates that the existence of deaf bands can explain the disagreement between experiments and theory.

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[1] E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).
[2] S. John, Phys. Rev. Lett. **58**, 2486 (1987).
[3] E.N. Economou and M. Sigalas, *Photonic Band Gaps and Localization*, edited by C.M. Sokoulis (Plenum Press, New York, 1993).
[4] M.M. Sigalas and E.N. Economou, J. Sound Vib. **158**, 377 (1992).
[5] M.S. Kushwaha, P. Halevi, L. Dobrzynski, and B. Djafrari-Rouhani, Phys. Rev. Lett. **71**, 2022 (1993).
[6] E.N. Economou and M. Sigalas, Phys. Rev. B **48**, 13 434 (1993).
[7] F.R. Montero de Espinosa, E. Jiménez, and M. Torres, Phys. Rev. Lett. **80**, 1208 (1998).
[8] J.V. Sánchez-Pérez, D. Caballero, R. Mártinez-Sala, C. Rubio, J. Sánchez-Dehesa, F. Meseguer, J. Llinares, and F. Gálvez, Phys. Rev. Lett. **80**, 5325 (1998).
[9] K.M. Ho, C.T. Chan, and C.M. Sokoulis, Phys. Rev. Lett. **65**, 3152 (1990).
[10] C.M. Anderson, and K.P. Giapis, Phys. Rev. Lett. **77**, 2949 (1998).
[11] D. Caballero and J. Sánchez-Dehesa (unpublished).
[12] R. Mártinez-Sala, J. Sancho, J.V. Sánchez, V. Gómez, J. Llinares, and F. Meseguer, Nature (London) **378**, 241 (1995).
[13] M.M. Sigalas and E.N. Economou, Europhys. Lett. **36**, 241 (1996).
[14] M.S. Kushwaha, Appl. Phys. Lett. **70**, 3218 (1997).
[15] W.M. Robertson and W.F. Rudy III, J. Acoust. Soc. Am. **104**, 694 (1998).